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Is the Planck Momentum Attainable?

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Abstract

We present evidence that an interplay of the laws of microphysics and cosmology renders the Planck momentum unattainable by an elementary particle. Several categories of accelerators are analyzed and shown to fail.

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1 Introduction

The Planck mass, $m_{Pl} = (\hbar c/G_{Newton})^{1/2} \approx 10^{19} \text{ GeV}$ and corresponding length $l_{Pl} = \hbar/m_{Pl}c \approx 10^{-33}cm$ or time $t_{Pl} = l_{Pl}/c$, are of fundamental importance, marking the onset of strong non-renormalizable quantum gravity effects. In this “superplanckian” regime the theoretical framework of local field theory and indeed the very concept of space time may break down. Many different lines of reasoning suggest that l_{Pl} and m_{Pl} are the minimal distance to which the location of an elementary particle can be defined and the maximal energy to which an elementary localized degree of freedom can be excited.

In the following we address the question of the highest energy that can be given to a single elementary particle in our universe subject to the known laws of microphysics and cosmology. As one approaches the Planck regime novel physics effects may come into play, modifying the very concept of an elementary degree of freedom. This new, fundamental, Planck scale physics may, in turn, directly prevent crossing the “Planck barrier”, i.e. prevents achieving $W \gg m_{Pl}$ for one elementary degree of freedom. However we are starting with the familiar low energy regime where weakly coupled theories, with degrees of freedom corresponding to local fields or pointlike quarks, leptons, photons, and other gauge bosons apply. Our question whether any device utilizing electromagnetism, gravity, or strong interaction (QCD) can accelerate such an elementary particle to an energy $W \geq m_{Pl} \approx 10^{19}GeV$ is therefore well posed. Interestingly we find that an interplay of microphysics and cosmological parameters may prevent acceleration to $W_{Pl} \approx 10^{19}GeV$

already at this level.

Strictly speaking accelerating one particle to $W = m_{Pl}$ causes only one, longitudinal, (z) dimension to shrink up to $\Delta z \leq l_{Pl}$. It does not, by itself, generate say energy densities in the superplanckian regime: $U \equiv \frac{dW}{dV} \geq m_{Pl}^4$. To achieve the latter in say a qq collision¹ we need to accelerate the two quarks in opposite directions so as to have an invariant center of mass energy $\sqrt{(P_1 + P_2)^2} \approx m_{Pl}$ and then both have to mutually scatter with a transverse momentum transfer $|\vec{q}| \approx m_{Pl}$ so as to achieve localization also in the transverse (x, y) directions down to $\Delta x, \Delta y \leq l_{Pl}$. The cross section for such a collision, $\sigma \approx l_{Pl}^2$, is extremely small ($\approx 10^{-66} cm^2$) making the goal of achieving superplanckian energy densities far more difficult than the mere acceleration of one particle to $W \geq m_{Pl}$.² We find however that even the latter goal seems to be unattainable. It should be emphasized that this does not stem from any kinematic limitation of say a maximal Lorentz boost. Thus boosts vastly exceeding $\gamma = 10^{19}$ (or $2 \cdot 10^{22}$) required to achieve Planck energy proton (or electron) are implicit in having very energetic photons. The latter can be viewed as very soft photons boosted by $\gamma \gg 10^{23}$.

The group property of Lorentz boosts implies that one very large boost can be achieved in many successive small steps. Thus consider the set of “Gedanken” nested accelerators illustrated schematically in Fig. (1). Sup-

¹In reality qq scattering is generated via pp collisions. The momentum fraction carried by the valence quarks evolves to zero as $Q^2 \rightarrow \infty$ but only logarithmically. Thus we can still achieve in principle the superplanckian qq collisions if the protons are accelerated to somewhat higher energies $m_{Pl} * \ln(m_{Pl}/\Lambda_{QCD})$.

²Naively one would think that if an accelerator capable of accelerating one proton to m_{Planck} can be built, there is no intrinsic difficulty in joining two such accelerators back to back to achieve CMS invariant Planck energy.

pose that a proton is accelerated inside some microscopic device D_1 (see Fig. (1)) to $\gamma = 11$. The whole D_1 device in turn sits within a larger setup D_2 , which boosts D_1 to $\gamma = 11$. D_2 in turn sits inside $D_3 \dots$ etc., etc. The device D_{18} is then also boosted by D_{19} to $\gamma = 11$, thus finally achieving transplanckian energies $E = \gamma^{19} GeV \approx 10^{20} GeV$ for our proton. It is hard to imagine a non-local mechanism impeding the operation of the huge accelerator D_{19} just because nineteen layers down, inside the innermost microscopic D_1 one proton is about to achieve Planck energy.

However strict Lorentz invariance applies only if space time is uniform and flat. Precisely due to gravity ($G_N > 0$) the universe is curved, and the horizon grows (since the big bang) only with a finite velocity $c < \infty$.

The idealized nested accelerator and many others which we discuss below are either larger than the present universe, are too fragile and breaks down, or are too compact and massive and collapses into a black hole. More sophisticated multi-stage devices are flawed by the amplification of tiny errors in the process of acceleration or require overly complex corrective feedback.

Clearly the issue here is not of overall energetics. Just one ton of matter moving at a modest velocity of Km/sec has kinetic energy $m_{Pl}c^2 \approx 10^{16} ergs$. This is also the output in one second of a high intensity laser. The real “bottleneck” is the focusing of all of this energy residing initially in 10^{30} nucleons or photons onto a single proton!

We proceed next to discussion of various type of “Gedanken accelerators”.

2 EM Accelerators

Electromagnetism presents an easily controlled long range force and hence it is most natural to consider its utilization for accelerators first. Electromagnetism, along with quantum mechanics and the mass of the electron, also fix the scale of atomic (chemical) energies and material strength. Terrestrial EM accelerators operating at limited ($R \leq 10km$) scales already produce $E_{proton} \approx 10^4 GeV$. Similar devices of cosmic dimensions could naively be envisioned with far larger acceleration energies. Indeed even naturally occurring cosmic ray acceleration generates primaries (protons!) with $E \approx 10^{12} GeV$ which have already been experimentally observed.

The claim that protons or electrons with energies $E_{Pl} \approx 10^{19} GeV$ can never be achieved is therefore far from obvious. Hopefully it will become clearer as we proceed through a list of more and more sophisticated accelerators.

3 Rocket Boosting

An obvious objection to the notion of maximal energy is its non-Lorentz invariant nature. Thus if we put our “laboratory” or the accelerator on a rocket and boost it by a factor γ_R the energy that we see in our lab will be enhanced by an extra factor of γ_R

Let us assume that the rocket is boosted via emitting “burnt fuel” at a velocity β relative to the rocket. If the latter ever becomes relativistic then

momentum conservation yields (henceforth $c = 1$)

$$\frac{\Delta m_R \beta}{\gamma_R} = \Delta p = \Delta \gamma_R m_R \quad (1)$$

with $\frac{\Delta m_R \beta}{\gamma_R}$ the red shifted momentum of the emitted mass element balanced by the increment $\Delta \gamma_R m_R$ of the rocket momentum. Eq. (1) readily yields

$$m_f = m_i e^{-\frac{\gamma_f^2}{\beta}} \quad (2)$$

Normal exhaust velocities are very small making m_f hopelessly small. Even if $\beta = 1$, i.e. the rest mass is converted into photons, just to achieve $\gamma_f \approx 10$ for a 100 kg “laboratory” requires $m_i \approx 10^{17} m_{sun}$! Thus no appreciable boost can be achieved and this naive realization of the “nested accelerators” scheme is impossible.

4 Acceleration via Photon Pressure

Instead of accelerating macroscopic pieces of matter, one may accelerate single charged particles via photon pressure. An intense laser beam can accelerate charged particle to high energies by repeated Compton scattering. As the particle approaches the putative super-planckian regime, it becomes extremely relativistic. In the particle’s rest frame the photons will be strongly red-shifted (by a γ^{-1} factor) and $\sigma = \sigma_{Thompson} \approx \pi \alpha^2 / m^2$ is appropriate. If the energy flux Φ_E of the photon beam is:

$$\frac{dW}{dt} = \Phi_E \sigma (1 - \beta) = \Phi_E \frac{8\pi \alpha^2}{3m^2} \frac{m^2}{W^2} \approx \Phi_E \frac{\alpha^2}{m_{Pl}^2} \frac{4\pi}{3} \quad (3)$$

The key factor limiting the rate of energy gain is the difference between the photon and particle velocities $(1 - \beta) \approx \frac{1}{2\gamma^2} = \frac{m^2}{2W^2}$.

Integrating we find that after time t ,

$$W_f = (4\pi\alpha^2)^{\frac{1}{3}}(\Phi t)^{\frac{1}{3}} \quad (4)$$

The flux of energy in the beam is related to the mean $\langle E^2 \rangle$ by

$$\frac{1}{4\pi} E^2 = \Phi. \quad (5)$$

We will demand that $E \leq E_{crit} \approx \frac{\pi m_e^2}{e}$ so as to avoid vacuum breakdown via e^+e^- pair creation[1]. Hence

$$\Phi \leq \frac{\pi m_e^4}{4e^2} \approx \frac{m_e^4}{16\alpha} \quad (6)$$

and

$$W_f \leq (\frac{\pi}{2}\alpha)^{1/3} m_e (m_e t)^{1/3} \approx GeV (l/cm)^{1/3} \quad (7)$$

where $l \approx t$ is the length of the accelerator. Evidently the growth of energy with linear dimension $l, \sim l^{1/3}$ is too slow. Even if $l = R_{Hubble} = 10^{28} cm$, $W_f \leq 2.10^9 GeV$. If the lightest charged particle were not the electron, but rather some heavier particle x , W_f would be larger by a factor of $(\frac{m_x}{m_e})^{4/3}$. Planck energies would still be unattainable so long as m_x does not exceed $10^5 GeV$!

5 Circular Accelerators

The energy loss due to synchrotron radiation for a circular accelerator is given by [2]

$$\frac{\delta W}{rotation\ period} = \frac{4\pi\alpha}{3} \frac{\gamma^4}{R}. \quad (8)$$

The mild requirement that δW should not exceed $W_{final} = \gamma_f m$ (namely that all the energy is not radiated away in one turn), implies

$$W_f \leq \left(\frac{4\pi}{3}\alpha\right)^{1/3} m(mR)^{1/3} \quad (9)$$

or $\gamma_f \leq (mR)^{1/3}$. Eq. (9) is fortuitously similar to Eq. (8) above with m_e replaced by m , the mass of the accelerating particle. Even for $m = m_{proton}$ and $R = R_{Hubble}$ ³

$$W_f \leq 4.10^{13} GeV. \quad (10)$$

Another important corollary which follows from the large rate of synchrotron loss (Eq. (8) above) is: Acceleration to an energy W along a circular arc of length L and an angle $\Theta = L/R$ is impossible unless

$$\theta \leq \left(\frac{WL}{\alpha}\right)^{1/2} \gamma^{-2}. \quad (11)$$

This relation will be useful in subsequent considerations.

6 Acceleration in 3-D Electromagnetic Fields

The simplest, “brute-force”, accelerator consists of a finite region of space of dimension R in all directions in which an electric field E exists. The Final energies obtained are limited by:

$$W = eER \quad (12)$$

³Again we note that if we had a stable, massive, larger particle x of mass $m_x \geq 10^6 m_{proton}$ the limit of Eq. (9) would appear to allow Planckian acceleration.

The “3-D” field in question could be due to two “capacitor” plates of size $R \times R$ placed at a distance R apart. In principle it appears that $eER \approx m_{Pl}$ can be obtained even for small E if R is sufficiently large. A more “realistic” setup could be an extreme rotating neutron star such that the intense B field generates an almost equal E field

$$|\vec{E}| = |\vec{\beta} \times \vec{B}| \leq |\vec{B}| \quad (13)$$

We will not address the important issues of how one builds cosmic capacitors or the effects of synchrotron radiation due to magnetic bending of particle trajectories in the magnetic field around the neutron star-which drastically reduces the maximal energies obtained. The interesting point is that regardless of any details no such three dimensional configuration can, in principle, achieve Planck energies.

The total energy stored in the B and E fields, $\frac{1}{2}[\int E^2 + B^2] \approx (B^2 + E^2)R^3$, should not exceed the critical value Rm_{Pl}^2 , or the whole system will collapse into a black hole of Schwarzschild radius R . However, $(B^2 + E^2)R^3 \leq Rm_{Pl}^2$ implies $ER \leq m_{Pl}$ (or $BR \leq m_{Pl}$) and since $e = \sqrt{4\pi\alpha} \leq 1$, we find that $W_{max} = eER$ (or eBR) $\leq m_{Pl}$, this result is independent of how compact and strong or extended and weak the field configuration may be. The only key requirement is that the electromagnetic charge of the elementary particle accelerated be smaller than unity.⁴

⁴The weakly coupled electron, rather than a putative corresponding strongly coupled monopole, with $g \simeq e^{-1} \gg 1$, should be considered as elementary. Then the t’Hooft-Polyakov monopoles are extended objects of size $\frac{1}{e} \frac{1}{m_M}$ and effectively contain $\approx \frac{1}{\alpha}$ elementary quanta.

7 Linear Accelerators

The excessive masses and synchrotron radiation can apparently be avoided in the case of linear accelerators. Existing LINACs have gains

$$G \equiv \frac{\Delta W}{\Delta z} \approx e\bar{E} \approx 10 \text{ MeV/meter} \quad (14)$$

and extend over lengths $L \approx$ few kilometers. Electron energies $W_e \approx 50 \text{ GeV}$ and $\gamma_e \approx 10^5$ can thus be achieved. Scaling up L and/or G one might hope to achieve $W = m_{Pl} \approx 10^{19} \text{ GeV}$.

Let us first note that the gain, G , is limited by

$$G \leq G_{max} = eE_{max} \approx \frac{\alpha^3 m_e^2}{2} \approx \text{Rydberg/Bohr radius} \quad (15)$$

E_{max} is the maximal electric field that can be generated or sustained by charges or currents in normal matter. Stronger fields would readily ionize hydrogen atoms. Such fields overcome the work function and skim all conduction (valence) electrons breaking all materials. In general any physical system that stores or guides energy with density $U/V = u$ experiences a pressure

$$p \approx \text{const.} u \quad (16)$$

with a coefficient of order unity. The maximal E corresponds then to the maximal pressure

$$p_{max} \approx E_{max}^2 \approx \alpha^5 m_e^4 \approx 10^{13} \text{ dynes/cm}^2 \quad (17)$$

that materials can withstand.

Even for $G_{max} \approx eE_{max} \approx eV/A^0 \approx 100 MeV/cm$ the minimal, “net”, acceleration length required for achieving energy W is:

$$L_{min} \equiv W/G_{max} \approx 10(W/GeV)cm (= 10^{20}cm \text{ for } W = m_{Pl}) \quad (18)$$

Further, to avoid local buckling under the pressure inside the pipe the thickness of the pipe, should be comparable with the radius d .

Let us consider next the electromagnetic field of the accelerating particle itself. The rest frame coulomb field

$$\vec{E}^{(0)} = e\hat{r}/r^2 \quad (19)$$

has a coulomb energy

$$W_{coul}^{(0)} \approx \int_d^{2d} \vec{E}^{(0)2} d^3\vec{r} \approx \alpha/2d \quad (20)$$

stored in the pipe wall. In the lab frame this becomes an electromagnetic pulse of duration shortened by Lorentz contraction down to

$$\Delta t \approx \frac{d}{\gamma} \quad (21)$$

and total energy

$$\Delta W = \gamma W_{coul}^{(0)} = \gamma\alpha/2d \quad (22)$$

According to the W.W. method[2] this pulse is equivalent to a pulse of real photons of energy ($\hbar = 1$)

$$\omega_\gamma \approx \frac{1}{\Delta t} = \frac{d}{\gamma} \approx \frac{\gamma 10^{-14} GeV}{d(in \text{ cm})} \quad (23)$$

The high energy equivalent photons will produce e^+e^- pairs on the wall nuclei with the Bethe-Heitler cross section

$$\sigma \approx (Z^2\alpha^3/m_e^2)ln(\omega_\gamma/m_e) \quad (24)$$

This leads to an energy loss at a rate:

$$\frac{\Delta W}{\Delta z}|_{Loss} = \Delta W n \sigma \quad (25)$$

It is convenient to parameterize the density of scattering nuclei via ($\hbar = 1$)

$$n = \xi(a_{Bohr})^{-3} = \xi(m_e \alpha)^{-3} \quad (26)$$

with the dimensionless ξ being typically $\approx 10^{-2}$. Demanding that the loss rate does not exceed the

maximal gain

$$\frac{\Delta W}{\Delta z}|_{Loss} \leq \frac{\Delta W}{\Delta z}|_{maxgain} = G_{max} \leq \frac{1}{2} m_e^2 \alpha^3 \quad (27)$$

yields then using (31) (32) (30) and (28):

$$\left(\frac{\gamma}{m_e d}\right) \ln\left(\frac{\gamma}{m_e d}\right) \leq \frac{\xi}{\alpha^3 Z^2} \quad (28)$$

Thus, for $Z \approx 30$, we find that unless

$$diameter \approx d \geq 10^8 cm \approx 10^3 km \approx thickness, \quad (29)$$

we will not be able to achieve $W = 10^{19} GeV$ for protons. This implies in particular that

$$M_{pipe} \approx 2\rho\pi d^2 L \geq 2.10^4 M_{Sun}(!) \quad (30)$$

The large length ($L \geq 10^{20} cm$) and diameter ($d \geq 10^8 cm$) required exacerbate another crucial difficulty; namely, that of guiding the accelerating particle to remain at all times along the central axis, and maintaining this linear

trajectory with very high precision. Such guidance is required for several reasons. The light-like particle suffers a gravitational deflection of order

$$\delta\theta_0 = gL = \frac{R_{Schw}}{R^2}L \quad (31)$$

as it travels a distance L in the field g of a galaxy of Schwarzschild radius R_{Schw} at a distance R away. Using $R \approx 2.10^{24}cm$ and $M_{Gal} = 10^{13}M_{Sun}$ as “typical” values ⁵ and $R_{Schw} = G_N M_{Gal} = 10^{13}R_{Schw.Sun} = 2.10^{18}cm$ we find by comparing with Eq. (8) that the radiative synchrotron losses, due to the gravitational bending will be excessive once $L \geq 10^{19}cm$, one order of magnitude smaller than $L_{min} \approx 10^{20}cm$.

More generally, let us envision some transverse “wobbling” of the accelerated particle around the axis with wavelength λ and amplitude δ . Using Eq. (11) (with $\Theta \sim \frac{\delta}{\lambda}, L \sim \lambda, W = \gamma m_N, \gamma \approx 10^{19}$) we find that in order to avoid excessive synchrotron losses we must satisfy

$$\delta \leq 3\lambda(\lambda m_N)^{\frac{1}{2}}10^{-28}. \quad (32)$$

Even if we take $\lambda \approx d = 10^8cm$, an incredible accuracy of trajectory with

$$\delta \leq 10^{-9}cm \quad (33)$$

should be maintained at all times. This would seem to be virtually impossible if we indeed maintain an empty pipe hole and guide the particle only via fields generated at the pipe wall at a distance of 10^8cm away.

A key observation in this context is that on-line monitoring and correction of the orbit is impossible. Thus let us assume that at some “station”

⁵These refer to the distance and mass of the Andromeda galaxy.

S_N we find the position and velocity of the accelerating particle and then transmit the information to the next “station” S_{N+1} “downstream” so that some corrective action can be taken there. If the distance between the two stations is ΔL then a signal traveling with the velocity of light will arrive only a very short time

$$\delta t = \frac{\Delta L}{c}(1 - \beta) = \frac{\Delta L}{2c\gamma^2} \quad (34)$$

ahead of our particle. The distance that a corrective device can move at that time is only

$$\delta = \frac{\Delta L \beta_D}{2\gamma^2} \quad (35)$$

where β_D is the velocity with which the corrective device moves.

Using $\gamma = 10^{19}$ we find (even for $\beta_D = 1$!) that

$$\delta \leq \frac{\Delta L}{10^{38}} \quad (36)$$

so that even for $\Delta L \approx L \approx 10^{20}cm, \delta \leq 10^{-18}cm!$

Our considerations of the linear accelerator touched only some of the possible difficulties. Yet these arguments strongly suggest that such a project is not merely difficult because of, say, the need to assemble a $10^{20}cm$ pipe of 20,000 solar masses. Rather there are inherent, “in principle”, difficulties which make the project of accelerating particles to Planck energy via a linear accelerator impossible.

8 Gravitational acceleration

Gravity is, in many ways, the strongest rather than the weakest interaction. This is amply manifest in the gravitational collapse to a black hole which

no other interaction can stop. Along with the very definition of the Planck mass, m_{Pl} , this naturally leads us to consider gravitational accelerators, and the acceleration (or other effects) of black holes in particular.

9 Direct Acceleration

If a particle of mass μ falls from infinity to a distance r from the center of a spherical object of mass m , it obtains, in the relativistic case as well, a final velocity

$$\beta_f = \sqrt{\frac{r_{SW}}{r}} \quad (37)$$

with $r_{SW} = \frac{G_N m}{c^2}$, the Schwarzschild radius of the mass m . In order that our particle obtain, in a “single shot”, planckian energies, we need that $\epsilon \equiv 1 - \beta_f = \frac{1}{2\gamma_f^2} = \frac{\mu^2}{2m_{Pl}^2}$ or $\epsilon = \frac{r - r_{SW}}{r_{SW}} = \frac{\mu^2}{2m_{Pl}^2}$. The last equation applies also if at infinity we have initially a photon or massless neutrino of energy μ . Using for the generic starting energy or rest mass $\mu \leq m_N \approx 1GeV$ we then find

$$\epsilon \equiv \frac{r - r_{SW}}{r_{SW}} \leq 10^{-38} \quad (38)$$

since to avoid trapping the last ratio should exceed 2 this Planck acceleration is clearly ruled out.

10 Hanging Laboratory

The difficulty of using the gravitational fields in the neighborhood of the black hole horizon is best illustrated by considering the following concept of a “hanging laboratory” (suggested to us by N. Itzhaki as a possible counterexample to the impossibility of achieving Planck energy). Thus let us

envision a black hole of very large radius R_{Schw} . We can then hang our laboratory keeping it at a distance d away from the horizon. If R is sufficiently large there will be minimal tidal distortion across the laboratory. An energetic particle falling towards our laboratory would then be blue-shifted by a factor R/d and can appear superplanckian to us, once $\frac{R}{d}$ is large enough.

Amusingly, this fails simply because any “rope” used to hang the laboratory and extending between aR_{Schw} and bR_{Schw} with $a \geq b \approx O(1)$ tears under its own weight. Imagine that the rope is “virtually displaced” downwards around the upper hanging area by some distance δz which we take to be a_0 , the inter-atomic distance in the rope material. This will decrease the gravitational energy by

$$\delta m \left(\frac{1}{a} - \frac{1}{b} \right) \approx O(\delta m) \quad (39)$$

with δm the mass of a rope element of length δz :

$$\delta m = S \rho \delta z = S (M(A, Z)/a_0^3) a_0 \quad (40)$$

where S is the cross-sectional area of the rope and $\rho = M(A, Z)/a_0^3$ the mass density with $M(A, Z) = Am_N$ the average mass of the nuclei in the rope material. A displacement by $\delta z = a_0$ causes the tearing of the rope. The energy required for tearing the rope is $N\epsilon$ with

$$N \approx S/a_0^2 \quad (41)$$

the number of atomic nearest-neighbor bonds torn along the z direction and $\epsilon \approx \frac{1}{2} m_e \alpha^2$ is our usual estimate for the bond energy. Tearing will be avoided

if the gravitational energy gained (which from Eq. (39) is $\approx \delta m$) cannot supply the $N\epsilon$ energy required i.e. if:

$$N\epsilon \geq \delta m \approx NM(A, Z) \approx NAm_N, \quad (42)$$

which leads to the condition (note that as a_0 cancels):

$$\frac{\alpha^2 m_e}{2m_N} = 1.4 \cdot 10^{-8} > A \geq 1 \quad (43)$$

This inequality clearly fails. We can also show that tearing cannot be avoided by having the cross section of the rope change so that a broader section on top can more readily sustain the weight of a lighter section on the bottom.

11 The Unruh Accelerator

A sophisticated accelerator using repeated “slingshot kicks” was suggested by Unruh. This beautiful concept is best illustrated in the following simple two black holes context. Consider first two black holes of equal mass $m_1 = m_2 = m$ at points P_1 and P_2 located at $+L, -L$ along the z axis. A relativistic neutral particle μ (neutron, neutrino, or photon) is injected parallel to the z -axis with some relative impact parameter near $z = 0$. With an appropriate choice of the impact parameter $b = b_0$, the accelerated particle describes a “semi-circle” trajectory around m_1 at P_1 , and is reflected around this mass by an angle $\theta = \pi$ exactly. Moving then along a reflected ($x \rightarrow -x$) trajectory the particle μ approaches the other mass m_2 at P_2 , and is reflected there by $\theta = \pi$ as well. The particle will eventually describe a closed geodesic trajectory bound to the two mass m_1, m_2 system. In reality these two masses

move. For simplicity consider the case when the masses move symmetrically towards each other with relative velocity β . Transforming from the rest mass of m_2 say to the “Lab frame”, we find that in each reflection the energy of μ is enhanced according to $W_\mu \rightarrow W_\mu \sqrt{\frac{(1+\beta)}{(1-\beta)}}$. The last equation represents the boost due to the “slingshot kick” alluded to above. If we have N such reflections, the total boost factor is $\sim (\frac{1+\beta}{1-\beta})^{\frac{N}{2}}$. In order to achieve Planck energies, for starting energies $\sim GeV$, this overall boost should exceed 10^{19} . However, in this simple geometry the total number of reflections is limited by $N = 1/\beta$. After more reflections, the two masses will either coalesce or reverse their velocities, leading now to a deceleration of the particle μ upon each reflection. Furthermore, in order to avoid the two masses coalescing into a black hole upon first passage, we find that β_{max} , the maximal β along the trajectory, is $\beta_{max} = \frac{2}{3\sqrt{3}}$. The total amplification is therefore bound by $(\frac{1+\beta_{max}}{1-\beta_{max}})^{\frac{1}{2\beta_{max}}} \leq 2.6$.

The Unruh set up involves, however, two additional heavier black holes $M_1 = M_2 = M$ with m_1, m_2 , revolving around M_1, M_2 respectively, in circular orbits of equal radius R and period $T = 2\pi R/\beta$, with β the orbital velocity. The two orbits are assumed to lie in the $(x - z)$ plane with the centers of the circles located at $(x, z) = (0, +L)$. The “top points” on the two circles, *i.e.* the points where x is maximal, define now the original reflection centers $P_1, P_2 = (R, -L), (R, +L)$. The oppositely rotating masses m_1 and m_2 are synchronized to pass at P_1 and P_2 , respectively, at the same time – once during each period T . Furthermore, the motion of the accelerated mass μ is timed so as to have μ at the extreme left point on its “Stadium Shaped”

orbit $(x, z) = (R, -L - \rho)$ or, at the extreme right point $(x, z) = (R, L + \rho)$, at precisely the above times. This then allows us to achieve the desired sling shot boosts, repeating once every period T . Note that $2T$ is now the period of the motion of the overall five body system $(M_1, M_2, m_1, m_2; \mu)$.

However, the inherent instability of this motion limits the number N of periods (and of sling shot boosts) and foils this ingenious device. The assumed hierarchical set-up $L \gg R \gg r_{SW} \approx b_0$ can be used to approximate the angular deflection of μ while it is circulating around m_1 , say, by

$$\theta = \int_{u_{min} \simeq 0}^{u_{max}} \frac{du}{\sqrt{\frac{1}{b^2} - u^2(1 - 2G_N m u)}} \quad (44)$$

with b the impact parameter and $u_{max} = 1/\rho$ corresponding to the turning point of closest approach. Independently of the exact (inverse elliptic function) dependence of θ on b/r_{SW} , ρ/r_{SW} , a fluctuation δb^0 around the optimal b^0 , for which θ equals π , causes a corresponding fluctuation in the reflection angle θ : $\delta^1 \theta = \pi - \theta = k \delta b^0 / b^0$, with the dimensionless constant c of order one. The large distance L transforms this small $\delta \theta$ into a new impact parameter deviation, $\delta^1(b) = L \delta^1 \theta$. The ratio between successive deviations of the impact parameter is then given by $|\delta^1(b)| = (cL/b_0) \delta^0(b)$, etc. After N reflections, we have therefore

$$\delta^N(b) \simeq (kL/b_0)^N \delta^{(0)}(b) \quad (45)$$

For $L > R > b_0$, the ratio in the last equation $cL/b_0 \gg 1$.

The individual sling-shot gain $\sqrt{\frac{1+\beta}{1-\beta}}$ depends on the linear velocity β of the circular motion of m_1 around $M_1 = M$ (or m_2 around $M_2 = M$). Let

R_{SW} be the Schwarzschild radius of the large mass M . For circular motion with radius R the above β is given by

$$\beta = [\frac{R_{SW}}{2(R - R_{SW})}]^{\frac{1}{2}} \quad (46)$$

In order to avoid capturing $m_{(i)}$ on $M_{(i)}$ we need that $R \geq \frac{3}{2}R_{SW}$ i.e. $\beta \leq \beta_{max}^{circ} \leq \frac{1}{\sqrt{2}}$. Since we like the motion of μ to be dominated by m_i and avoid its (μ 's) falling towards the large masses $\frac{R_{SW}}{R}$ and β_{max} have to be much larger. However even with the above β_{max} we need $N \approx 54$ repeated sling shot kicks to achieve an overall $[(1+\beta_{max})/(1-\beta_{max})]^{\frac{N}{2}} \approx 10^{19}$ enhancement. To avoid complete orbit deterioration for the accelerating particle μ , *i.e.* to avoid $\delta^N(b) \simeq b_0$, we need then, according to Eq. (45), even for a modest $(cL/b_0) \approx 10$, an initial precision $\delta b^0/b_0 = 10^{-50}$, which, in particular, is exceeded by the quantum uncertainty in b . (This beautiful refutation of the Unruh accelerator is due to B. Reznik.)

The above Unruh accelerator is actually a prototype of many other gravitational accelerators in which the high energy is achieved by repeating many stages of more limited boosts. Another example, suggested to us by A. Polyakov, involves the phenomenon of super-radiance[3]. Specifically if two particles fall towards a rotating black hole, and collide in its vicinity then one can be emitted with an energy higher than the total energy. It turns out again that the ratio of $E_{final}/E_{initial} \leq 1 + \epsilon$ with $\epsilon \approx 0,2$ so that many repeated such collisions with a series of Kerr black holes is required and again exponentially growing fluctuations are encountered.

Thus, ab-initio, finely tuned, perfect, gravitational accelerators are impossible. Could we still achieve such energies if we monitor the trajectory and

correct deviations? Since the accelerated particle is neutral (to avoid synchrotron losses), the correction of the trajectory requires additional mass(es). Just adding such masses only complicates the problem turning the Unruh accelerator into say, a six (or more) body problem. Hence we need two ingredients: (i) extra non-gravitational forces to navigate the corrective masses and (ii) means for monitoring the trajectory of the accelerated particle. Neither of these tasks seems readily achievable. Thus to have an appreciable light bending effect the corrective mass should be large - probably a black hole itself and its propulsion via non-gravitational forces (say some rocket mechanism) appears impossible

The second task of monitoring the location and momentum $\vec{r}(t), \hat{p}(t)$ of the accelerated particle μ is even more difficult. To this end we need to scatter other particles from μ . Since, among others, we need to correct for the effect of quantum fluctuations we have to monitor each accelerated particle individually, by having several scatterings from μ during one traversal of its closed orbit.

Such scatterings, being intrinsically quantum mechanical, introduce further, uncontrolled, perturbations. Furthermore, these scatterings systematically deplete the energy of the accelerated particle μ . For concreteness take $\mu = \gamma$ (photon) and assume we scatter electrons from it. In any $e\gamma$ collision, be it elastic or inelastic, the initial photon retains only a fraction of its energy.⁶

⁶This follows essentially from kinematics: when the energetic photon scatters “elastically” on an electron at rest it retains its direction in the Lab frame. However if in the center of mass the scattering is by an angle θ^* , then the ratio of the final and initial photon energies, in the Lab, is $\frac{W'}{W} \approx \frac{1+\cos\theta^*}{2}$. Since in the center of mass frame the

12 Evaporating Black Holes

Hawking evaporation of black holes naturally leads to single quanta of energies approaching (but not exceeding!) m_{Pl} . Hawking found[4] that a black hole has an effective temperature $T_{BH} \approx \frac{1}{R_{BH}} \approx \frac{m_{Pl}^2}{m_{BH}}$ with R_{BH} , m_{BH} the radius and mass of the black hole. As R_{BH} approaches l_{Pl} , the temperature T_{BH} approaches m_{Pl} , and photons, or other quanta, with energies $\approx T_{BH} \approx m_{Pl}$ could, in principle, be emitted. However, precisely at this point, also the total mass of the black hole approaches m_{Pl} and energy conservation forbids the emission of several such quanta or one quantum with $W \gg m_{Pl}$.

If as it emits the last quanta, the center of mass of the black hole had an appreciable boost, say $\gamma \geq 3$, then the quanta emitted in the direction of motion of the black hole could be Doppler shifted and have superplanckian energies: $W' = \gamma W_{inblackholeframe} \geq m_{Pl}$. The recoil momentum accumulated through the Hawking radiation is, at all stages, $P_{Rec} \approx m_{Pl}$ ⁷, so that $\gamma_{Rec} \sim 1$, and no appreciable extra boost effect is expected. Note that $P_{Rec} \approx m_{Pl}$ implies a recoil kinetic energy of the black holes $W_{recoil} \approx \frac{P^2}{2M_{BH}} = \frac{m_{Pl}^2}{2M_{BH}} = T_{BH}$ as required by equipartition.

In principle, the black hole can be directly boosted so that the extra Klein-Nishima formula for the Compton process yields roughly an isotropic distribution we have $\langle \frac{W'}{W} \rangle \approx \frac{1}{2}$.

In passing we note that if the particles with which the accelerating particle μ collides have similar energy and (for $m_\mu \neq 0$) similar mass then the elastic collision yields in the final state a more energetic and a slower particle.

⁷This amusing result is very simply explained. Consider the overall recoil momentum accumulated when the black hole loses half its initial mass via $N \approx \frac{m_{BH}}{T_{BH}} \approx (\frac{m_{BH}}{m_{Pl}})^2$ quanta. The “random” vectorial addition of the N recoil momenta yields $|\vec{P}_{Rec}| = |\vec{K}_1 + \dots \vec{K}_N| \approx \sqrt{N}K = \sqrt{N}T_{BH} \approx m_{Pl}$.

Doppler shift yields transplanckian energies for the last photons. In reality this boost is rather difficult to achieve. If the black hole is to evaporate in Hubble time its initial mass and radius cannot exceed $m_{BH}^{(0)} \approx 10^{15}g$ and $R_{BH}^{(0)} \approx 10^{-13}cm$ respectively. Thus even a single electric charge on the black hole creates a field $eE \approx \frac{\alpha}{R_{BH}^2}$ which exceeds the vacuum breakdown limit $eE \approx m_e^2$ by a factor of about a hundred already for the initial radius. Hence the black hole will immediately lose its charge via vacuum e^+e^- pair creation and electromagnetic acceleration is impossible.

Finally, gravitational boosting of the mini black holes is excluded by the reasoning presented in Sections (9),(11). In particular since the minimal distance to which the mini black hole can approach the big black hole without being captured onto it, is limited by r_{Schw} , the Schwarzschild radius of the large black hole, we cannot achieve even $\gamma = 2$ boosts.

13 Can Small Black Holes Be Created?

Another serious difficulty with using evaporating mini black holes as Planck accelerators - albeit for $O(1)$ quanta emitted at the last stage - stems from the fact that such black holes cannot be created ab-initio in the lab. The bottleneck is again the need to focus excessive energy onto a tiny domain of size R_{Schw} - the Schwarzschild radius of the prospective black hole.

To see that, consider the following “attempt” to build small black holes by focusing energy. Imagine a spherical arrangement of N high intensity lasers each of cross section $\approx d^2$ and wavelength λ on a large spherical shell of radius L . It is designed to focus the energy emitted by the lasers which

are all directed radially inward into a hot internal spot of diameter d . The number of lasers N is restricted by

$$N \leq 4\pi\left(\frac{L}{d}\right)^2 \quad (47)$$

Let the energy density in each laser beam be $\rho_0 \approx E^2/2$. We can achieve N -fold enhancement of this density in the central spot of radius d

$$\rho_{center} \approx N\rho_0 = NE^2/2 \quad (48)$$

This generates a black hole of radius d provided that

$$M_{(inside\ d)} = \frac{4\pi d^3 \rho_{center}}{3} = \frac{2\pi}{3} NE^2 d^3 \geq 2m_{Pl}^2 d \quad (49)$$

or

$$\sqrt{N}Ed \geq \sqrt{\frac{3}{\pi}}m_{Pl}. \quad (50)$$

Diffraction limits the size Δ of the spot image of any of the individual laser apertures according to:

$$\Delta \geq \frac{L}{d}\lambda \approx \sqrt{\frac{N}{4\pi}}\lambda. \quad (51)$$

Demanding that Δ not exceed the original aperture d which is also the assumed black hole radius implies then

$$N \leq 4\pi d^2/\lambda^2 \quad (52)$$

The upper bound on E , required to avoid vacuum breakdown

$$E \leq m_e^2/\sqrt{\alpha} \text{ (or } m_e^2\alpha^{\frac{5}{2}}) \quad (53)$$

and Eqs. (50) and (52) imply

$$\sqrt{\frac{3}{\pi}}m_{Pl} \leq \sqrt{N}Ed \leq \sqrt{4\pi}\frac{d^2}{\lambda^2}\frac{m_e^2}{\sqrt{\alpha}} \quad (54)$$

or

$$d \geq \sqrt{\frac{3}{\pi}} \left(\frac{m_{Pl}}{m_e} \right)^{\frac{1}{2}} \alpha^{\frac{1}{4}} \sqrt{\frac{\lambda}{m_e}} \approx 10^{10} \sqrt{\frac{\lambda}{m_e}} \quad (55)$$

demanding that the evaporation time of our black hole be shorter than the Hubble time implies

$$d^3 m_{Pl}^2 \leq R_{Hubble} \approx 10^{28} cm. \quad (56)$$

If we use this along with

$$d^3 m_{Pl}^2 \geq 10^{30} \frac{m_{Pl}^2}{w_x^{\frac{3}{2}} m_e^{\frac{3}{2}}} \quad (57)$$

(with $w_x = \frac{1}{\lambda_x}$ the photon's energy) which readily follows from Eq.(55) we conclude that

$$W_\gamma \geq 10^{20} GeV \quad (58)$$

That is we need super-Planckian photons to start with! There is the “standard” mechanism of generating black holes via the collapse of supermassive ($m_{core} \geq (2 - 3)m_{Chandrasekhar} \approx m_{BH}$) stellar cores with $m_{Chandrasekhar} \approx \frac{m_{Pl}^3}{m_N^2} \approx 1.4M_\odot$, the Chandrasekhar mass. The evaporation time of such a black hole is:

$$t_{evap} \approx \left(\frac{m_{Pl}}{m_N} \right)^6 t_{Pl} \approx 10^{70} sec (\approx 10^{53} t_{Hubble}). \quad (59)$$

It has been speculated by Lee Smolin [[5]] that the final decays of these black holes will spawn new universes.

14 The “Inverted Cascade Accelerator” Utilizing Repeated Particle Collisions

Most of the limitations and bounds on accelerators that were found above stem from the fact that our starting particles have some limited energy ⁸, E_0 and that normal materials have limited energy density and strength $\approx E_0^4$. We may try to generate an “inverted cascade” process where fewer and fewer particles will survive at consecutive stages but with increasing energy.

$$E_k = \lambda_k E_{k-1}; \quad \lambda_e \geq 1 \tag{60}$$

Since the starting energy in each stage is higher, the original energy-strength bounds do not apply. This is a crucial difference between this Gedanken accelerator and the, superficially similar, rocket accelerator. There even later stages are constructed from ordinary fragile material. Indeed we do not envision here a static accelerator, but rather a sequence of transient or “single-shot” devices. It is modeled in an abstract way after the schematic “nested accelerator” of Fig. (1). We are, however, building consecutively the various stages of the accelerator starting from the biggest outermost stage first as the very acceleration process proceeds. In this “dynamical accelerator” earlier stages which have fulfilled their mission literally “evaporate” and disappear.

Let us assume that the process starts at the zeroth stage with N_0 particles, each with kinetic energy E_0 , so that the total initial energy is

$$W_0 = N_0 E_0 \tag{61}$$

⁸ In this section and the next we will use E_0, E_k , etc. to denote the energy of individual particles and W_0, W_k , the energy of the complete system with N particles

In the k th stage we have N_k particles of average energy E_k , and the total energy at this stage is

$$W_k = N_k E_k \quad (62)$$

If no “waste heat energy” is dissipated in going from one stage to the next then $W_k = W_0$ and after K stages all the energy would concentrate in one single particle of energy

$$E_K = \prod_{k=1}^K \lambda_k E_0 = N_0 E_0 = W_0 \quad (63)$$

However such a process - where all the many, low energy final, particles conspire to reconstitute the energetic primary proton is completely impossible. Specifically, we need to worry about the second law of thermodynamics as well.

Thus along with a fraction N_{k+1}/N_k of the particles of the k th generation which have been elevated to higher energies $E_k \rightarrow E_{k+1} = \lambda E_k$ and constitute the $(k+1)$ th stage of the accelerator, we need to emit a certain minimal amount of energy as “waste heat” in the form of particles of energy lower than E_K . What is the maximal efficiency of the k th stage accelerator, i.e. what is the maximal value of the ratio

$$\epsilon_k \equiv \frac{W_{k+1}}{W_k} = \frac{N_{k+1} E_{k+1}}{N_k E_k} ? \quad (64)$$

Let us identify the average energy of a particle in the k th stage with a fictitious “effective temperature” T_k for this stage.

$$T_k = E_k. \quad (65)$$

Let us follow the pattern of our schematic idealized nested accelerator of Fig. (1). Thus we identify the k th accelerating stage as a device which transfers a fraction ϵ of its energy to the next, $k + 1^{th}$ stage and at the same time dissipates the rest. The maximal thermodynamic efficiency of such a device is limited by:

$$\epsilon_k \leq (2 - T_k/T_{k+1})^{-1} \quad (66)$$

or

$$\epsilon_{k-1} \leq \frac{T_k}{T_{k+1}} \left(1 - \frac{T_{k+1} - 2T_k + T_{k-1}}{T_{k+1}}\right)^{-1} \quad (67)$$

Hence the equation is bound by

$$\epsilon \equiv \Pi \epsilon_k \leq \frac{T_0}{T_k} \prod_{k=1}^K \left(1 - \frac{T_k''}{T_{k+1}}\right)^{-1} \approx \frac{T_0}{T_k} = \frac{E_0}{E_{max}} \quad (68)$$

where T_k'' indicates a second derivative with respect to k and we assumed that we have many stages with $\delta T_k \equiv T_{k+1} - T_k \ll T_k$

In Eq. (68) $E_{max} \approx E_k$ is the final, maximal, energy achieved. To achieve E_{max} starting with particles of energy E_0 , we need to have initially at least $((E_{max}/E_0)/\epsilon)$ particles i.e. $N_0 \geq (E_{max}/E_0)^2$. This result is independent of the amplification λ and the corresponding total number of stages K required so long as K is sufficiently large and the enhancement ratios λ_k sufficiently close to one so as justify the approximation used in Eq. (68).

We have not succeeded in constructing (even Gedanken!) mechanical or EM inverted cascade accelerators with the above efficiency. Roughly speaking the energetic particles in the k^{th} stage tend to disperse transversally. A long range, coherent, attractive force seems to be required in order to keep these particles confined. Gravity can precisely supply that. Indeed the

evaporating black hole of section 9 can be viewed as an “inverted cascade” Planck accelerator. To this end we should view its initial stage with mass $W_0 = M_{BH}^{(0)} = R_{BH}^{(0)} m_P^2$ and corresponding temperature $T_{BH}^{(0)} \approx \frac{1}{R_{BH}^{(0)}}$ as a collection of $N^{(0)}$ Hawking photons each with average energy $T_{BH}^{(0)}$ so that all together we have

$$N_0 = \frac{W_0}{T_0} \approx (R_{BH} m_{Pl})^2 \approx \left(\frac{R_{BH}}{l_{Pl}}\right)^2 = \left(\frac{m_{Pl}}{T_0}\right)^2 \quad (69)$$

In the process of Hawking radiation some of these “photons” are radiated away - the total energy decreases, but the remaining, fewer, photons get “hotter” i.e. more energetic according to: $E \approx T \approx \frac{1}{R_{BH}} \approx \frac{m_{Pl}^2}{W}$ until ultimately we stay with $O(1)$ Planck photons $W_{final} = E_{final} \approx m_{Pl}$, and Eq. (69) precisely conforms to the above $N_0 = (E_{max}/E_0)^2$ with $E_{max} = m_{Pl}$.⁹

15 A Conjecture on Maximal Energy of Accelerators

The accelerators considered so far - with the exception of the evaporating black hole - fall into two basic categories: accelerators where a large number of quanta of some common low energy E^0 are absorbed by the accelerating particle, and those involving predesigned, classical field configurations. The latter include neutron stars, the Gedanken capacitor fields and also the linear accelerator. Much energy is stored in advance in coherent collective classical degrees of freedom. These degrees of freedom then directly interact with and

⁹The optimality of the black hole in “upgrading” energy up to a Planckian level may be related to another interesting issue of maximal number of elementary “computations” in a given spacetime region. (S. Massar and S. Popescu, work in preparation.)

accelerate the particle.

The photon beam accelerator of section 4 is typical of the first category. For all accelerators in the first category there is a clear-cut limit on the rate of energy increase. It follows from the confluence of the limited energy density or energy flux $\Phi \leq E_0^4$ and the $(1 - \beta) \approx \frac{1}{2\gamma^2}$ relative velocity factor. If E_0 is some effective low energy physics mass scale (= energy of photons, atomic energies, nuclear energies, nucleons mass,...) then this implies

$$\frac{d\gamma}{dt} \leq \frac{E_0}{2\gamma^2} \quad (70)$$

and t_{final} the time required to achieve γ_{final} is therefore

$$t_f \geq \frac{\gamma_f^3}{E_0} \quad (71)$$

The issue of acceleration via coherent preexisting fields is less clear. The energy density in the accelerating field is still limited by $u \leq E_0^4$ with E_0 some low energy physics scale. In addition to this we also need to address the questions of how the classical field is to be generated in the first place and also of the stability of the trajectory of the accelerating particle. We would like to speculate that given all these limitations the general bound Eq. (71) still applies.

It is amusing to apply this speculation to super-high cosmic rays. Recently such primary cosmic rays, with energies exceeding $3 \cdot 10^{11} \text{ GeV}$ have been observed[6][7]. These findings appear to conflict with the Greisen-Zatsepin-Kuzmin bound on cosmologically originating primary protons (because of the energy degradation by the 3° background radiation). However Eq. (69) (with a “Natural” choice $E_o \simeq m_N \simeq \text{GeV}$) would allow, in Hubble

time, γ_f values up to

$$\gamma_f^{max} = (R_{Hubble} \cdot GeV)^{\frac{1}{3}} = 10^{14} \quad (72)$$

still exceeding the maximal value observed by ≈ 100 .

Needless to say the maximal cosmic ray energy observed to-date need not be the true absolute end of the cosmic ray spectrum. This raises an interesting issue. What if protons (or neutrons) of energy $\geq 10^{14}$ GeV are ever found and our speculated upper upper bound value (Eq. (69)) indeed applies? We would then be forced to the radical conclusion that these protons/neutrons must originate from the decays of long lived particles x of mass $m_x \geq 10^{15}$ GeV.

16 Hubble Time and Mass Dependence of the Maximal Energy Achievable

Certain Planck accelerators have been ruled out by the fact that the Hubble radius of the universe is too small to accommodate them. Present observations and estimates of $\Omega(\equiv \frac{\rho_{cosmic}}{\rho_{critical}})$ tend to favor an open, or critical, universe which keeps expanding forever. It would seem therefore that after waiting a sufficiently long time ($t \approx 10^{15} t_H$ according to the scaling law of Eq. (71) with $E_0 \approx 1 GeV$ acceleration to super-Planckian energies becomes feasible.

We observe, however, that truly cosmic accelerators are “red-shifted”, in the process of the expansion. This red shift reduces the rate of energy increase $\frac{dW}{dt}$ in such a way that even waiting for an infinite time will not enhance the final energy obtained by more than a factor of order one.

To see this, consider, as an example, the photon beam accelerator of Section (4). Here $\frac{dW}{dt} \approx \frac{\Phi\alpha^2}{W^2}$ decreases with the expansion, simply because Φ , the energy flux in the beam red-shifts according to

$$\Phi(t) = \Phi(t_H)[a(t_H)/a(t)]^4 \quad (73)$$

with $a(t)$ the scale factor and $t_H \approx 10^{19} \text{sec}$, the “present” time. Using $a(t) = a(t_H)(t/t_H)^{\frac{2}{3}}$ - appropriate for a matter dominated universe - we then have

$$\frac{dW}{dt} = \frac{\Phi(t_H)\alpha^2}{W^2} \left(\frac{t_H}{t}\right)^{\frac{8}{3}} \quad (74)$$

which integrates to

$$W^3(t) - W^3(t_H) = \frac{9}{5}\Phi_0\alpha^2 t_H \left[1 - \left(\frac{t_H}{t}\right)^{\frac{5}{3}}\right] \quad (75)$$

Thus, waiting for $t = \infty$ rather than for $t = 2t_H$ will enhance the final energy gathered in the waiting period only by $[1 - (\frac{1}{2})^{\frac{5}{3}}]^{-1}$ i.e. by $\leq 50\%$. In passing we note that since at present $a(t) \approx t$ implying that in a few t_H 's the universe will be curvature dominated and $a(t)$ starts growing at a faster, linear, rate. Also considering Eq. (71) we note that even drastic reduction of the red-shifting of $\frac{dW}{dt}$ to $\frac{dW}{dt} = \frac{\Phi(t_H)\alpha^2}{W^2} \left(\frac{t_H}{t}\right)^p$ with $p \geq 1$ still will allow only a $(\ln \frac{t}{t_H})^{\frac{1}{3}}$ increase of W_{final} .

Our statement that the horizon is too small really means that the dimensionless Dirac number: $Di = R_{Hubble}m_e(m_N) \approx 10^{37} - 10^{40}$ is in some sense “small”. In particular it does not much exceed or is smaller than other dimensionless combinations that naturally arose in our previous discussions

such as $N_0 = (\frac{m_{Pl}}{E_0})^2 \approx (\frac{m_{Pl}}{m_e})^2$ or $(\frac{m_{Pl}}{m_N})^2 = 10^{38} - 10^{40}$, the minimal number of particles in an ideal inverted cascade accelerator or $N'_0 = (\frac{m_{Pl}}{E_0})^3 \approx 10^{57} - 10^{66}$ which arises in the discussion of section 12. If m_{Pl} is left fixed, which we assume to be the case, we can enhance the Dirac combination relative to N_0, N'_0 by enlarging E_0 . This leads to the amusing question of whether Planck accelerators would be feasible in a hypothetical case where the proton/electron masses are increased say by a common factor λ .¹⁰

Indeed as noted above (Sections (4) and (5)) this would reduce synchrotron radiation and enhance the rigidity of materials and of the vacuum against e^+e^- pair production breakdown in strong electric fields. Hence taking $\lambda \gg 1$ appears to help facilitate super-Planckian accelerators. Clearly the cosmology of such a Gedanken universe is likely to drastically change - the enhanced gravitational interactions may lead to quick recollapse drastically decreasing by as much as a factor λ^{-1} or even λ^{-2} the maximal t_H (or R_H).

It turns out however that physics on much shorter scales is also drastically modified, preventing dramatic increase in the maximal energy achieved by accelerators. The simple scaling $m_e \rightarrow m'_e = \lambda m_e$ but $\alpha \rightarrow \alpha' \approx \alpha$ will scale down by λ atomic and lattice unit sizes. From Eq. (17) above we find that the new materials will be able to withstand much larger maximal pressure

$$p_{max} \propto \alpha^5 m_e^4 \rightarrow p'_{max} \approx \alpha^5 m_e'^4 \approx \lambda^4 p_{max} \quad (76)$$

and for this reason so will be the gain in say the linear accelerator

¹⁰This can be achieved if all lepton, quark, and Λ_{QCD} scales are scaled up by a common λ . This can be done leaving gauge couplings almost the same up to mild “running” (i.e. renormalization group) logarithmic changes.

$$G' = \frac{dW'}{dx'} \approx \alpha^3 m_e'^2 = \alpha^3 m_e^2 \lambda^2 = \lambda^2 G \quad (77)$$

Thus an original LINAC capable of achieving maximum energy $W_{max} = GL$, can now achieve $W'_{max} = G'L' = \lambda W_{max}$ even if L' is scaled down (with all other dimension) by λ^{-1} .

In general the pressures in materials due to self gravity are negligible unless the dimensions are large. If we have a uniform body of size R (in all directions) then the gravitational pressure is

$$p_G = \frac{F_{Grav.}}{Area} \approx \frac{G_N M^2 / R^2}{R^2} = \frac{G_N M^2}{R^4} \quad (78)$$

If we use $\rho = GeV/a_0^3 \approx m_N/a_0^3 = m_N m_e^3 \alpha^3$ for ordinary materials and $M = \rho R^3$ then: $p_G = G_N \rho^2 R^2 = \frac{m_N^2 m_e^6 \alpha^6 R^2}{m_{Pl}^2}$ It exceeds $p_{max} = \alpha^5 m_e^4$ when $R \geq 10^{10} cm$ and hence hypothetical large cold stars of such dimensions would liquify at the center. For a given R p_G scales with λ^8 whereas the maximal pressure Eq. (76) only with λ^4 . Hence the maximal size of new material is $R \leq (\frac{10^5}{\lambda})^2 cm$. Thus taking $\lambda = 10^5$ (so as to allow bridging the gap between our conjectured maximal energy W_{max} (in EM accelerators) $\approx 10^{14} GeV$ and m_{Pl}) will make even $(cm)^3$ devices crush - leaving little room for Planckian accelerators.¹¹

¹¹Note that we do not claim that the present parameters of elementary particles are optimally chosen so as to avoid or to facilitate Planck acceleration - though the earlier version of this paper contained some speculation that rare Planckian collisions led to universes with different, lighter, Fermionic generations. We found that Lee Smolin[5] has indeed speculated that black holes do give rise after Hawking evaporation to baby universes, and that the fundamental physical parameters are such that the rate of black hole formation and breeding of new universes is maximized.

17 Summary, Comments and Speculations

The above discussion strongly suggests that elementary particles with super-Planckian energies may be unachievable. Is this indicative of new physics or just a curiosity? There is the well known example of our inability to build a Heisenberg microscope so as to beat the uncertainty principle. However, unlike in the celebrated case, we do not (yet!) see a single common principle causing all our Gedanken accelerators to fail.

It has been conjectured that Planck scale physics can manifest in the low energy regime by inducing effective interactions which violate all global symmetries. An example is a $\frac{\lambda}{m_{Pl}}\Phi^+\Phi\Phi^+\Phi\Phi$ term where the Φ bosons carry two units of lepton number. Such a term violates $U(1)_{B-L}$. It endows the putative massless Goldstone boson (Majoron), associated with a spontaneous breakdown of this Global $U(1)$, with a finite mass. A concrete mechanism for B-L violation involves the formation of a black hole in a collision of, say, $\Phi^+\Phi$, followed by the decay of the B.H. into $\Phi\Phi\Phi^+$, a final state with two units of lepton number. In this way the violation of the global quantum numbers traces back to the fundamental “No Hair Theorem” for black holes. Exactly as in the case of $SU(5)$, where a virtual X, Y GUTS meson can mediate nucleon decay by generating effective four Fermi terms, the virtual “mini black hole” system was conjectured to induce the $\frac{\lambda}{m_{Pl}}\Phi^+\Phi\Phi^+\Phi\Phi$ term. The estimated resulting Majoron mass $M_x \approx KeV$ is rather high.[8] Also Planckian black holes would constitute some irreducible environment and may require modification of quantum mechanics

However our inability to achieve super-Planckian energies could be due to

some profound principle. In the “ultimate” theory the whole super-Planckian regime may then be altogether excluded - much in the same way that in quantum mechanics the simultaneous definition of x and p to better than $\Delta x \Delta p \leq \hbar/2$ is impossible. All these global quantum number or quantum mechanics violating effects will then not be there - and super Planck physics even in terms of its indirect low energy manifestations could be completely absent. Building a theory of this kind is an outstanding challenge that clearly will not be attempted here.

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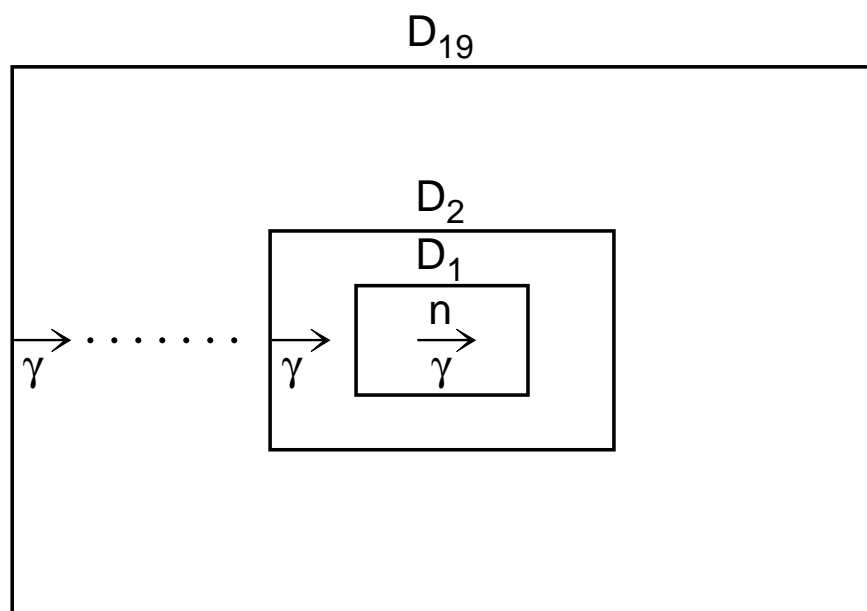


Figure 1: Accelerator within accelerator within... system designed to achieve super-Planck energies.

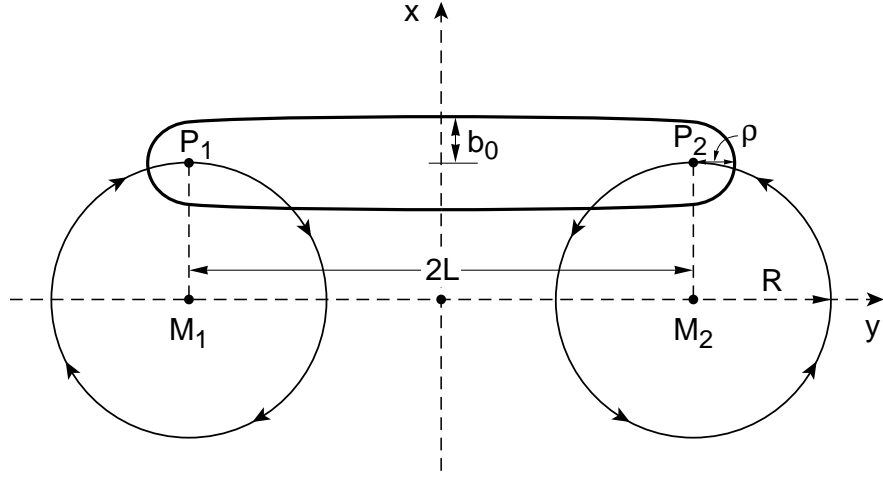


Figure 2: The Unruh accelerator. The two black holes $m_1 = m_2 = m$ go around the stationary more massive $M_1 = M_2 = M$ black holes in circular orbits of radius R and in opposite directions. The accelerating particle goes around in the oblong “stadium-like” trajectory of thickness $2b_o$ with b_o the impact parameter. It gets the “sling-shot kicks” boosting its energy as it goes around P_1, P_2 at times $t, t + T$ with m_1, m_2 at P_1, P_2 respectively.